

**2.1 Key Terms/Concepts:**

Tangent

Secant Line

Average Rate of Change

Instantaneous Velocity

**2.1 Formulas**

Average Rate of Change/Average Velocity

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y(t_2) - y(t_1)}{t_2 - t_1} = \frac{\Delta \text{position}}{\Delta \text{time}}$$

**Section 2.1 p. 87 #3 (modified)**

The point  $P = (1, \frac{1}{2})$  lies on the curve  $y = x / (1 + x)$

(a) If  $Q$  is the point  $(x, x / (1 + x))$ , find the slope of the secant line PQ when  $x$  is:

(i) 0.9

(ii) 0.99

(iii) 1.1

(iv) 1.001

(b) From (a), guess the value of the slope of the tangent line.

(c) From (b), find the equation of the tangent line at the point  $P$ .

**Section 2.1 p. 87 #6 (modified)**

A rock is thrown upward on the planet Mars with a velocity of 10 m/s and its height (position) at  $t$  seconds later is described by  $y = 10t - 1.86t^2$ .

(a) Find the average velocity over the given time intervals:

(i) [1, 1.5]

(ii) [1, 1.1]

(iii) [1, 1.01]

(iv) [1, 1.001]

(b) Estimate the instantaneous velocity when  $t=1$ .

**2.2 Key Terms/Concepts:**

Limit of a Function at a Point

Left-Hand (LH) Limits

Right-Hand (RH) Limits

Limit exists if and only if RH &amp; LH limits are equal in value

Vertical Asymptote at point  $a$ **2.2 Formulas –what does each mean?**

$$\lim_{x \rightarrow a^-} f(x) = L$$

$$\lim_{x \rightarrow a^{(\pm)}} f(x) = \pm\infty$$

$$\lim_{x \rightarrow a^+} f(x) = L$$

$$\lim_{x \rightarrow a} f(x) = L$$

$$\lim_{x \rightarrow a} f(x) = L \text{ iff } \lim_{x \rightarrow a^+} f(x) = L \text{ and } \lim_{x \rightarrow a^-} f(x) = L$$

**Section 2.2 p. 97 #5**Use the graph of  $f$  in the book to state the value of each if it exists. If it does not, explain why:

(a)  $\lim_{x \rightarrow 1^-} f(x)$

(b)  $\lim_{x \rightarrow 1^+} f(x)$

(c)  $\lim_{x \rightarrow 1} f(x)$

(d)  $\lim_{x \rightarrow 5} f(x)$

(e)  $f(5)$

**Section 2.2 p. 98 #14**Sketch the graph of what the function  $f$  may look like under the given conditions:

$$\lim_{x \rightarrow 0^-} f(x) = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = -1$$

$$\lim_{x \rightarrow 2^-} f(x) = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = 1$$

$$f(2) = 1$$

 $f(0)$  is undefined**Section 2.2 p. 98 #26**Determine the infinite limit. *Hint: There are two ways of doing this: try a table and do the limiting process as in Section 2.1 OR you can try graphing it*

$$\lim_{x \rightarrow -3^-} \frac{x+2}{x+3}$$

**2.3 Key Terms/Concepts:**

Limit Laws

Direct Substitution Law

Squeeze Theorem

**2.3 Formulas—what does each mean?**

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$$

**Section 2.3 p.107 #12**

Evaluate the limit if it exists.

$$\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4}$$

**Section 2.3 p. 107 #20**

Evaluate the limit if it exists.

$$\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$$

**Section 2.3 p. 107 #36**If  $2x \leq g(x) \leq x^4 - x^2 + 2$  for all  $x$ , evaluate  $\lim_{x \rightarrow 1} g(x)$ .

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$\lim_{x \rightarrow a} [f(x)]^n = \left[ \lim_{x \rightarrow a} f(x) \right]^n \text{ for } n \text{ any rational number}$$

$$\lim_{x \rightarrow a} c = c$$

$$\lim_{x \rightarrow a} f(x) = f(a) \text{ if } a \text{ is in the domain of } x$$

$$\text{If } f(x) \leq g(x) \leq h(x) \text{ and } \lim_{x \rightarrow a} f(x) = L = \lim_{x \rightarrow a} h(x), \\ \text{then } \lim_{x \rightarrow a} g(x) = L$$

**2.5 Key Terms/Concepts:** *Give a pictorial**example of each term*

1. Continuous
2. Discontinuous
3. Removable discontinuity
4. Infinite discontinuity
5. Jump discontinuity
6. Continuous from right/left at  $a$
7. Continuous on an interval
8. Intermediate Value Theorem

**Section 2.5 p. 128 #6 (modified)**

Sketch the graph of a function that has a jump discontinuity at  $x = 2$ , and a removable discontinuity at  $x = 4$ , and an infinite discontinuity at  $x = -1$ .

**Section 2.5 p. 129 #39**

Find the numbers at which  $f$  is discontinuous. At which of these numbers is  $f$  continuous from the right, from the left, or neither. Sketch the graph of  $f$ .

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } x > 1 \end{cases}$$

**Section 2.5 p. 129 #46**

Suppose  $f$  is continuous on  $[1, 5]$  and the only solutions of the equation  $f(x) = 6$  are  $x = 1$  and  $x = 4$ . If  $f(2) = 8$ , explain why  $f(3) > 6$ .

**2.6 Key Terms/Concepts:**

Horizontal Asymptote

**2.6 Formulas –what does each mean?**

$$\lim_{x \rightarrow \pm\infty} f(x) = L$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

**Section 2.6 p. 141 #20**

Find the limit.

$$\lim_{t \rightarrow -\infty} \frac{t^2 + 2}{t^3 + t^2 - 1}$$

**Section 2.6 p. 141 #32**

Find the limit.

$$\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{5 - 2x^2}$$

**Section 2.6**

Find the limit.

$$\lim_{x \rightarrow -\infty} \frac{\sin^2(x) \cos(x)}{1 - \cos^2(x)}$$

**Section 2.6**

Find the limit.

$$\lim_{x \rightarrow \infty} \frac{x^3 - 1}{x^3 + 2x - 185}$$

**2.7 Key Terms/Concepts:**

Tangent line slope  
 Difference Quotient  
 Instantaneous velocity  
 Derivative of a function at  $a$

**2.7 Formulas –What does each mean?**

$$\frac{f(x+h) - f(x)}{h}$$

$$\lim_{x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \rightarrow x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

**Section 2.7 p. 150 #7 modified**

Find the equation of a tangent line to the curve at the given point.

$$y = \sqrt{x} \text{ at } (4, 2)$$

**Section 2.7 p 151 #28**

Find  $f'(x)$ .

$$f(x) = \frac{x^2 + 1}{x - 2}$$

**Section 2.7 p. 151 #32, 33**

The following limits represent the derivative of some function  $f$  at some number  $a$ . State such  $f$  and  $a$ .

$$\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$$

$$\lim_{x \rightarrow 5} \frac{2^x - 32}{x - 5}$$

**2.8 Key Terms/Concepts:**

Differentiable at  $a$   
 Differentiable on an interval  
 Differentiation operators  
 Implications differentiation on continuity  
 Not Differentiable (3 cases)  
 Higher Derivatives(acceleration, jerk)

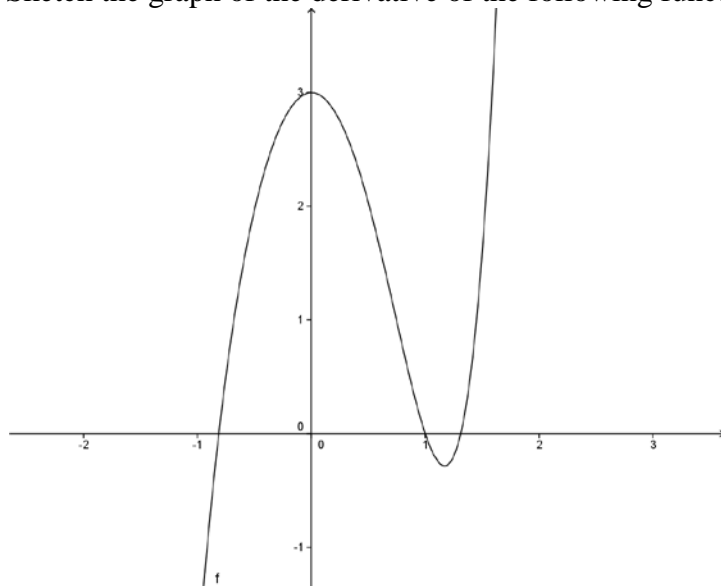
**2.8 Formulas –what does each mean?**

$$\frac{d}{dx}$$

$$\frac{d^2 s}{dt^2} = a, \frac{d^3}{dx^3}, \frac{d^{(n)}}{dx^{(n)}}$$

**Section 2.8 p. 162 #4-11 (modified)**

Sketch the graph of the derivative of the following function:

**Section 2.8 p. 163 #23**

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative:

$$f(x) = x^3 - 3x + 5$$